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Abstract

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Keywords: Cointegration tests; Power properties; Common-factor restrictions.

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ECM TESTS FOR COINTEGRATION IN A SINGLE EQUATION FRAMEWORK (*)

by

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Keywords: Cointegration, error correction models, power, common factor restrictions

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ABSTRACT

This paper proposes a new test for cointegration in a single-equation framework where the regressors are weakly exogenous for the parameters of interest. The test is denoted as ECM test and is based upon the OLS coefficient of the lagged dependent variable in an autoregressive-distributed lag model augmented with leads of the regressors. The limit distributions of the standardised coefficient and t-ratio versions of the ECM tests are obtained and critical values are provided. These limit distributions do not depend upon nuisance parameters but they depend on the number of regressors. Finally, we compare their power properties with those of other cointegration tests available in the literature and find under which circumstances the ECM tests have a better performance.

KEYWORDS. Cointegration tests; power properties; common-factor restrictions.

1. INTRODUCTION

This paper proposes a new test for cointegration in a single-equation framework. The new test is based on the coefficient of the lagged dependent variable in an autoregressive-distributed lag (ADL) model advocated by Hendry and Richard (1982) and Hendry (1987). This class of models has traditionally been used in the empirical literature to seek a tentatively adequate data-characterisation that encompasses rival models, displays parameter constancy, has martingale difference errors with respect to a selected information set and parsimoniously orthonormalises the regressors. As proved by Engle *et al.* (1983), *weak exogeneity* of the regressors for the parameters of interest is a sufficient condition for ordinary least squares (OLS) to provide asymptotically efficient estimates of the parameters in the conditional ADL model.

Recent papers by Phillips and Loretan (1991), Saikkonen (1991) and Hendry (1994) have extended the previous analysis to the case where regressors are $I(1)$ processes. A feature common to all of these papers is that they concentrate on the case of cointegration among the variables, proposing new methods to achieve asymptotically efficient estimates of the elements of the cointegrating vector. This strategy, which consists of adding leads of the regressors and the error correction term to the conditional model, has proved quite successful since, in contrast to the fully modified estimator of Phillips and Hansen (1990), where a semiparametric correction is needed, the correction of the OLS estimator and the computation of the final estimator are accomplished simultaneously in the time domain.

In this paper, instead of operating under the alternative hypothesis of cointegration, we choose to work under the null hypothesis of non-cointegration. We derive a test for cointegration denoted the error correction mechanism (ECM) test, which benefits from some of the advantages of ADL models described above. The procedure depends upon the significance of the lagged dependent variable since this is equivalent to testing the significance of the error correction terms in the ECM reparameterisation of the model. This type of test has been previously suggested by Banerjee *et al.* (1986), Banerjee *et al.* (1993) and Boswijk (1991). However, an extensive study of its properties is not yet available in the literature.

The ECM test, both in its normalised bias and t-ratio versions, has a limit distribution that does not depend on nuisance parameters. However, it is not dimension-invariant since its limit distribution shifts with the number of regressors. Alternatively, Hansen (1990) has proposed a cointegration test based upon the Cochrane-Orcutt (1949)

estimation procedure whose limit distribution is dimension-invariant and follows the unit-root distribution simulated by Fuller (1976). Nevertheless, this latter test, along with other well-known cointegration statistics such as the Engle and Granger (1987) test, suffers in finite samples from imposing potentially invalid *common factor* restrictions. Consequently, if these restrictions are not satisfied, the two latter types of test may have poor power properties. Since the ECM test does not suffer from this problem, there may be large advantages in its use.

Furthermore, as a by-product of the power analysis undertaken under a sequence of local alternatives of cointegration, we show that the t-ratio form of the ECM test may have better power properties than the normalised bias form, particularly when the common factor restrictions are grossly violated.

Lastly, it is important to note that, although the framework of reference is restricted to single equation conditional error correction models with a potentially unique cointegrating relationship, empirical studies abound where this is shown to be the case, e.g., money demand equations, consumption equations, etc.; cf. Hendry *et al.* (1984). Therefore, we believe that the applicability of the ECM test in applied work may be quite important. We therefore provide critical values based upon the limit distribution of the test for a wide number of regressors.

The outline of the rest of the paper is as follows. Section 2 presents the data generation process (DGP) for the simplified case where the regressors are assumed to be strictly exogenous, and derives the limit distributions of the ECM test-statistics under the null hypothesis of non-cointegration. Section 3 gives the corresponding limit distribution under a sequence of alternatives representing near-non-cointegration. Section 4 offers a comparison of the ECM test with other cointegration tests often used in applied work, stressing the problem of imposing possibly invalid common factor restrictions. Section 5 considers generalisations of the ECM tests to more realistic cases, where the regressors are only considered to be weakly exogenous. Section 6 provides Monte-Carlo finite-sample evidence about the relative performance of the ECM tests with respect to the other cointegration tests discussed in the paper. Finally, concluding remarks are given in section 7.

In common with most of the literature in this field, we follow some notational conventions. The symbol " \Rightarrow " denotes weak convergence of probability measures; " \rightarrow " denotes convergence in probability ; " \equiv " denotes equality in distribution; $BM(\Omega)$ refers to a Brownian motion with long-run covariance

matrix Ω ; $x_T = o_p(\phi_T)$ denotes that the sequence of random variables $\{x_T\}$ is of smaller order in probability than ϕ_T . Arguments of functionals on the space $[0,1]$ are frequently suppressed such that $\int_0^1 B^2(r) dr$ are written as $\int B^2$ to reduce notation. Proofs of important results are relegated to an appendix.

2. A SIMPLE DGP AND THE ECM TEST-STATISTIC

By using a simple DGP, based upon a single-equation error ECM model, this section describes the ECM testing procedure.

This DGP has been used elsewhere [c.f. Hendry and Richard (1982), Banerjee *et al.* (1993) and Kremers *et al.* (1992)] and has the form

$$\Delta y_t = \alpha \Delta x_t + \beta (y_{t-1} - \lambda' x_{t-1}) + \epsilon_t \quad (1)$$

$$\Delta x_t = u_t, \quad t = 1 \dots T \quad (2)$$

where α , λ and x_t are $(k \times 1)$ vectors of parameters and explanatory variables. The regressand y_t is a univariate process and β is a scalar; the initial conditions are, without loss of generality, set to zero and T is the sample size. The elements of x_t correspond to different regressors. The more general case where lags of Δx_t and Δy_t are allowed will be considered below. For the time being, we will assume that x_t is *strictly exogenous*, so that

$$\begin{matrix} 1 \\ k \end{matrix} \begin{pmatrix} \epsilon_t \\ u_t \end{pmatrix} \sim \text{i.i.d.} \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \Sigma_u \end{pmatrix} \right] \equiv \text{i.i.d.} (0, \Sigma)$$

where $\Sigma_u > 0$ to avoid cointegration among the regressors (for a brief discussion of the possibility of cointegrated regressors, see below). With this set-up, the partial sum processes $S_t(r) = T^{-1/2} \sum_1^{\lfloor Tr \rfloor} (\epsilon_t, u_t)'$ satisfy the multivariate invariance principles [c.f. Phillips and Durlauf (1986)].

$$S_T(r) \Rightarrow \Sigma^{1/2} B(r) \equiv BM(\Sigma)$$

where $B(r) = (B_\epsilon(r), B_u(r))'$ is a $(k+1)$ vector standardised Brownian motion i.e. $BM(I)$.

We further assume that $-2 < \beta \leq 0$. In this DGP, y_t and x_t are cointegrated when $-2 < \beta < 0$, while they are non-cointegrated when $\beta = 0$. Thus, tests of cointegration must

rely upon some estimate of the parameter β . Under the simplifying assumption that x_t is strictly exogenous, non-linear least squares (NLS) can be applied to (1) yielding consistent and asymptotically efficient estimates of α , β and λ . The ECM test statistic for cointegration, as suggested by Banerjee *et al.* (1986) and Boswijk (1991), is based upon estimating (1) by NLS and testing $H_0: \beta=0$. Alternatively, Banerjee *et al.* (1993), drawing upon results from Kiviet and Phillips (1992), show that a parameter-free distribution for the estimator of β can be achieved if x_{t-1} is added to (1), which is then estimated by OLS. This is so since, under the alternative hypothesis of cointegration, the true cointegrating slope λ is implicitly estimated when x_{t-1} is included as an additional regressor. Hence, according to this procedure, β is estimated by OLS from the unrestricted dynamic model

$$\Delta y_t = \alpha \Delta x_t + \beta y_{t-1} + \theta' x_{t-1} + \epsilon_t = \alpha \Delta x_t + \pi' w_{t-1} + \epsilon_t \quad (1')$$

where $w_t = (y_t, x_t)$ and $\pi = (\beta, \theta)'$.

Since $\beta(1, -\lambda)' = \pi'$, the non-cointegrating restriction $\beta=0$ implies $\pi=0$ and so the ECM test can be based upon the OLS estimator of β in (1') or on its t-ratio, denoted $\hat{\beta}_E$ and t_E , respectively. Thus, letting y and Δy be $(T \times 1)$ vectors of observations on y_t and Δy_t , the ECM estimator and its t-ratio are defined by

$$\hat{\beta}_E = [y_{-1}' M y_{-1}]^{-1} y_{-1}' M \Delta y \quad (3)$$

and

$$t_E = [\hat{\sigma}_\epsilon^{-2} y_{-1}' M y_{-1}]^{1/2} \hat{\beta}_E \quad (3')$$

where $M = I - V(V'V)^{-1}V'$ and V is a $(T \times 2k)$ matrix of observations on $v_t = (\Delta x_t', x_{t-1}')$, $\hat{\sigma}_\epsilon^2 = T^{-1} \sum_1^T \hat{\epsilon}_t^2$ and $\hat{\epsilon}_t$ is the OLS residual in (1').

Then, the following proposition holds

Proposition 1. For DGP (1)-(2) under the null hypothesis of non-cointegration ($\beta=0$)

$$T \hat{\beta}_E \Rightarrow [\int \bar{B}_\epsilon^2]^{-1} \int \bar{B}_\epsilon dB_\epsilon$$

and

$$\text{with } \bar{B}_\epsilon = B_\epsilon - (\int B_u B_u')' (\int B_u B_u')^{-1} B_u \blacksquare$$

$$t_E \Rightarrow [\int \bar{B}_\epsilon^2]^{-1/2} \int \bar{B}_\epsilon dB_\epsilon$$

Note that \bar{B}_ϵ is the residual from the continuous time regression of B_ϵ on B_u . Thus, although the previous limit distributions are free of nuisance parameters, they depend upon the number of elements (k) in x_t , as reflected by the presence of B_u in \bar{B}_ϵ , implying that corresponding test-statistics are not dimension-invariant.

3. DISTRIBUTION OF THE ECM TEST-STATISTICS UNDER THE ALTERNATIVE HYPOTHESIS OF COINTEGRATION

The alternative hypothesis is that of cointegration which, for DGP(1)-(2), is given by $-2 < \beta < 0$. Because the error-correction term in (1) is stationary under the alternative hypothesis, distributional results from conventional central limit theorems, instead of functional central limit theorems, apply for fixed alternatives. In contrast, under a suitable sequence of local alternatives, the non-conventional asymptotic theory developed by Phillips (1987, 1988) for near-integrated time series can be applied to sharpen the results on the asymptotic power of the ECM tests.

To proceed with the analysis of local power we consider the following parameterisation of the β coefficient

$$\beta = 1 - \exp(c/T) \simeq -T^{-1} c \quad (4)$$

In (4), c is a fixed scalar. We call time series that are generated by DGP (1)-(2), with β as in (4), near-non-cointegrated processes, following the terminology introduced by Phillips (1988) for univariate processes. The scalar c represents a non-centrality parameter which may be used to measure deviations from the null hypothesis $H_0: \beta=0$. When $c>0$, (4) represents a local alternative to H_0 , so that the rate of approach is controlled and the effect of the alternative hypothesis on the limit distribution of the test-statistics, based on the previous DGP, is directly measurable in term of the non-centrality parameter c .

To develop the analysis of local power, it is also useful to define the following disturbance

$$e_t = (\alpha - \lambda)' u_t + \epsilon_t \quad (5)$$

such that, under the previous assumptions about u_t and ϵ_t , $E(e_t^2) = \sigma_\epsilon^2 + (\alpha - \lambda)' \Sigma_u (\alpha - \lambda)$

Then, use is made of the following diffusion process

$$K(r) = \int_0^r \exp [c(r-s)] dB(s) \equiv B(r) + c \int_0^r \exp [c(r-s)] B(s) ds \quad (6)$$

associated with the standardised disturbances ϵ_t , u_t and e_t , denoted K_ϵ , K_u and K_e , respectively. Note that if $c=0$, then $K \equiv B$.

Using (5) and (6), it is possible to show the following result.

Proposition 2. For DGP (1)-(2) and (8), under the alternative hypothesis of near-non-cointegration ($c > 0$).

$$T \hat{\beta}_E \Rightarrow -c + (\sigma_\epsilon / \sigma_e) [\int \bar{K}_e^2]^{-1} \int \bar{K}_e dB_\epsilon \equiv \Phi_E$$

$$t_E \Rightarrow [\sigma_\epsilon^{-2} \sigma_e^2 \int \bar{K}_e^2]^{1/2} \Phi_E$$

$$\text{with } \bar{K}_e = K_e - (\int B_u K_e)' (\int B_u B_u')^{-1} B_u \blacksquare$$

Since $\sigma_e K_e = (\alpha - \lambda)' \Sigma_u^{1/2} K_u + \sigma_\epsilon K_\epsilon$, note that, when $c=0$, the non-centrality parameters of the two test-statistics are zero, i.e., $K \equiv B$ and the distributions under the null in Proposition 1 are recovered, i.e., power equals size.

Although the comparison of the asymptotic distributions under the local alternative hypothesis is cumbersome, given the complexity of the Wiener functionals derived above, some results can be obtained using the relationship in (5). To illustrate the main result, let us simplify the analysis by assuming that there is a single regressor, i.e., $k=1$. Then, given the relationship between e_t , ϵ_t and u_t that we repeat for convenience

$$e_t = (\alpha - \lambda) u_t + \epsilon_t$$

we will define a *signal-to-noise* ratio $q = (\alpha - \lambda)s$, with $s = \sigma_u / \sigma_\epsilon$, corresponding to the ratio of the (square root of the) variance of $(\alpha - \lambda)u_t$ relative to ϵ_t . This ratio will play a

prominent role in the analysis since, as $q \uparrow \infty$, it allows for "small- σ " approximations, i.e., $s^{-1} \uparrow 0$; cf. Kadane (1970,1971) Making use of this definition, the following proposition holds.

Proposition 3. For DGP (1)-(2) and (4), when $k=1$, under the alternative hypothesis of near-non-cointegration ($c>0$)

$$T\hat{\beta}_E \Rightarrow -c + o_p(q^{-1})$$

and

$$t_E \Rightarrow -c(1+q^2)^{1/2} (\int \bar{K}_e^2)^{1/2} + (\int \bar{K}_e^2)^{-1/2} \int \bar{K}_e dB_\epsilon + o_p(q^{-1}) \blacksquare$$

Various interesting properties arise from Proposition 3. First, asymptotically, as $q \uparrow \infty$, i.e., $\alpha \neq \lambda$ and $s \uparrow \infty$, the ECM test based upon the normalised bias has a slope equal to (minus) unity; since the limit distribution of $T\hat{\beta}_E$ is independent of q under the null and degenerates around $(-c)$ under the local alternative, the lower 5% tail of the distribution under the null will tend to be to the left of $(-c)$. Thus, we should observe very low power of the test based upon the coefficient when q is large. Second, the limit distribution of the ECM test based upon the t-ratio has a stochastic slope that depends upon q and does not degenerate around a single value, as is the case of the test based upon the normalised bias. Thus, when q is sizeable, the power of the t-ratio test will be greater than that of the normalised bias test. This is an interesting result, since, as shown by Phillips and Ouliaris (1990), under a fixed alternative hypothesis, the normalised bias test has non-centrality which grows at rate T while the non-centrality parameter of the t-ratio diverges at rate $T^{1/2}$. However, as shown in section 6, for reasonable sample sizes, the power of the t-test may be greater if q is sufficiently large.

4. COMPARISON WITH OTHER TEST-STATISTICS FOR COINTEGRATION

Among the already very large collection of cointegration tests available in the literature (c.f. Banerjee *et al.*, 1993), we wish to compare the power properties of the ECM test statistics with those of two popular test-statistics for cointegration in a single-equation framework. These are the Engle-Granger (1987) test-statistic and Hansen's (1990) Cochrane-Orcutt test-statistic. In what follows, we will denote these tests as EG and CO, respectively.

As is well known, the EG test is based upon a two-step procedure. In the first step a static OLS regression of y_t on x_t is implemented, yielding an estimate of λ , say $\hat{\lambda}$. Next, the cointegration test is based upon the normalised bias or the t-ratio of β in the regression

$$\Delta y_t - \hat{\lambda}' \Delta x_t = \beta (y_{t-1} - \hat{\lambda}' x_{t-1}) + \tilde{e}_t \quad (7)$$

where \tilde{e}_t is an error term such that $\tilde{e}_t = e_t + o_p(1)$

The CO test is similar in spirit to the EG test, except that the estimation of λ and the cointegration test are accomplished simultaneously, by estimating

$$\Delta y_t - \lambda' \Delta x_t = \beta (y_{t-1} - \lambda' x_{t-1}) + e_t \quad (8)$$

by NLS and testing for the significance of the NLS estimate of β . Hansen (1990, Theorem 2) proves that the normalised bias and the t-ratio, denoted as $T\hat{\beta}_\infty$ and t_∞ have the limit Dickey-Fuller distributions under the null hypothesis of non-cointegration. Thus, this test has the advantage over the ECM and EG statistics that its limit distribution is independent of the dimension of the vector x_t , a feature which according to Hansen (1990) may improve its relative power properties.

Nonetheless, as pointed out by Kremers *et al.* (1992) and Hansen (1995), both the EG and CO test suffer from the problem of imposing possibly invalid *common-factor* restrictions. This problem can be readily reviewed by considering the alternative representation of equation (1)

$$\Delta y_t = \alpha' \Delta x_t + \beta (y_{t-1} - \lambda' x_{t-1}) + e_t = \lambda' \Delta x_t + \beta (y_{t-1} - \lambda' x_{t-1}) + e_t \quad (9)$$

with e_t defined as in (5). As an extreme example, let $\sigma_e^2 \uparrow 0$ but $\alpha \neq \lambda$ and $(\alpha - \lambda)' \Sigma_u (\alpha - \lambda)$ is "substantial". In that case, the ECM regression has a near perfect fit with α , β and λ being estimated with near-exact precision, and the ECM test-statistics will be (arbitrarily) large. However, since $\sigma_e^2 = (\alpha - \lambda)' \Sigma_u (\alpha - \lambda) + \sigma_\epsilon^2$, the estimation of β in the CO and EG procedure will be much more imprecise, having an adverse effect on the power of both tests. In other words, where $\alpha \neq \lambda$, invalid common-factor restrictions are imposed in the estimation procedure underlying the latter tests, a feature which could have serious adverse effects on their power properties.

5. GENERALIZATIONS OF THE ECM TEST-STATISTICS

In the previous sections, we have assumed for simplicity that the vector of regressors x_t was strictly exogenous in the conditional model (1). However, this is a very strong assumption. As proved by Engle *et al.* (1983), all what is needed for OLS to be an asymptotically efficient estimation method for the parameters in (1') is that the x_t 's are weakly exogenous for the parameters of interest $\psi = (\alpha', \beta', \theta')$. This weaker assumption is fairly well used in practice (cf. Hendry *et al.*, 1984) and allows for the presence of lags of Δx_t and Δy_t in the conditional model (1). To extend the ECM test-statistics to this more general set-up, we will consider an extended DGP consisting of the following ADL conditional model

$$\gamma(L)\Delta y_t = \alpha(L)\Delta x_t + \beta(y_{t-1} - \lambda x_{t-1}) + \epsilon_t \quad (1'')$$

where $\gamma(L)$ and $\alpha(L)$ are polynomials in the lag operator L ; $\gamma(L)$ is a scalar polynomial of order m_γ and $\alpha(L) = (\alpha_1(L), \dots, \alpha_k(L))$ is a vector-polynomial of order (m_1, \dots, m_k) .

The marginal process for Δx_t is as in (2) with u_t being now a stationary process with zero mean and continuous spectral density $f_{uu}(\omega)$, whose covariance function is absolutely summable. In this more general framework, the partial sum process constructed from the $(k+1)$ vector $v_t = (\epsilon_t, u_t)'$ will now converge to a vector Brownian process $BM(\Omega)$ with long-run covariance matrix given by

$$\Omega = \begin{pmatrix} \omega_{\epsilon\epsilon} & \omega_{\epsilon u} \\ \omega_{u\epsilon} & \Omega_{uu} \end{pmatrix} = \Sigma + \Lambda + \Lambda' = \Delta + \Lambda' ; \quad \Sigma = \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \Sigma_u \end{pmatrix}$$

where $\omega_{\epsilon\epsilon} = \sigma_\epsilon^2$, $\omega_{u\epsilon} = E(\epsilon_o u_o') + \sum_{s=1}^{\infty} E(\epsilon_o u_s' + \epsilon_s u_o')$ and $\Omega_{uu} = E(u_o u_o') + \sum_{s=1}^{\infty} E(u_o u_s' + u_s u_o')$.

Under the assumption of weak exogeneity of x_t , we have it that $E(\epsilon_t/x_{t-1})=0$ for all $i \geq 0$. Thus, $\Delta_{21} = \sum_{s=0}^{\infty} E(u_o \epsilon_s) = 0$, but there is no guarantee that $\sum_{s=0}^{\infty} E(\epsilon_o u_s') = 0$. This would be implied by the stronger assumption that x_t is strictly exogenous for equation (1). Since $\Delta_{21}=0$, the "second-order" biases stressed by Phillips and Hansen (1990) will be absent in the distribution of the ECM test-statistics, as in Proposition 1. However, note that in this more general case B_ϵ and B_u are no longer independent Brownian motions. To illustrate this feature, take the following example.

For $k=1$, let $\Delta x_t = u_t = \gamma \Delta y_{t-1} + \eta_t = \gamma(\alpha u_{t-1} + \epsilon_{t-1}) + \eta_t$ with $E(\epsilon_t \eta_s) = 0$ for all t and s . Then, x_t will be weakly exogenous for the parameters of interest in (1), but the long-run covariance between B_ϵ and B_u will be $\gamma(1-\alpha\gamma)^{-1} \sigma_\epsilon^2$, under the null hypothesis of non-cointegration. This is so since $\sum_{k=0}^{\infty} E(\epsilon_o u'_k) \neq 0$, implying that the limit distributions obtained in Proposition 1 will now depend on nuisance parameters ($\omega_{u\epsilon}$). Hence the corresponding tests will not be asymptotically similar. Therefore, in principle, the computation of critical values in this more general case is problematic.

To overcome the problem of lack of similarity, we follow the strategy proposed by Phillips and Loretan (1991) and Saikkonen (1991) of correcting for serial correlation by augmenting the conditional model in (1'') with future values of Δx_t . Given the stationarity of u_t one would expect that the very remote future values of Δx_t only have a negligible impact on Δy_t and can therefore be ignored.

Under the previous conditions on the error terms, we may write

$$\epsilon_t = \sum_1^{\infty} a_j u_{t+j} + \xi_t \quad (10)$$

where $\sum_1^{\infty} ||a_j|| < \infty$ and ξ_t is a stationary process such that $E(\xi_t u'_s) = 0$ for all t and s .

Since the sequence $\{a_j\}$ in (10) is absolutely summable, we have it that $a_j=0$ for $|j|>S$ and S large enough. Thus, the ECM statistics may be computed from the model

$$\gamma(L) \Delta y_t = \alpha(L) \Delta x_t + \beta y_{t-1} + \theta' x_{t-1} + \sum_{j=1}^S a_j \Delta x_{t+j} + \xi_t \quad (11)$$

with $\xi_t = \xi_t + \sum_{|j|>S} a_j \Delta x_{t+j}$ in agreement with the assumption previously used by Said and Dickey (1984) we shall assume that

$$S^3/T \rightarrow 0 \quad \text{and} \quad T^{1/2} \sum_j ||a_j|| \rightarrow 0 \quad \text{for} \quad |j| > S$$

Using similar arguments to those in Saikkonen (1991, Theorem 4.1) it is straightforward to show that the limit distributions of $T\hat{\beta}_E$ and t_E , on the basis of regression model (11), are identical to those derived in Proposition 1. In practice the value of S should be so large that the coefficients a_j are effectively zero for $|j|>S$ while, at the same time, the least-squares estimation (11) is not feasible if S is too large compared with the sample size. In empirical applications, some experimentation with a

few values of S is advisable. Although a thorough discussion of this issue is beyond the scope of this paper, some experimentation along the lines of Stock and Watson (1993) seemed to suggest that the choice $S=1$ or 2 for $T=100$ had good size properties.

Next, it is important to note that, although deterministic terms have been ignored in the previous analysis for the sake of simplicity, the data may be demeaned, or demeaned and detrended, before applying the ECM tests for cointegration. The limit distributions of the various tests discussed in the paper in such cases are of the same form as in Proposition 1, except that Brownian motions are replaced by the appropriate Brownian bridges. Given the advantages of using the t -ratio for the ECM test, as discussed in section 4, the asymptotic critical values for the ECM t -ratio are reported in Table 1 up to five regressors. In order to analyse the finite sample distribution of those tests, critical values for four different sample sizes ($T=25, 50, 100$ and 500) are also presented. Since, as discussed in the Introduction, there are many examples in applied work of single equation conditional models with weakly exogenous regressors for the parameters of interest, we think that the above critical values may be widely applicable.

It is also noteworthy that the common factor problem of the EG and CO test-statistics for cointegration remains in this more general set-up. Furthermore, the Augmented Dickey Fuller (ADF) version of the EG test and the semi-parametric version proposed by Phillips and Ouliaris (1990) do not solve the problem. Since this argument is similar to that given by Kremers *et al.* (1992) where the potential cointegrating vector is assumed to be known *a priori*, we will simply summarise it briefly in what follows.

For example, if we consider the conditional model (1''), the error term in the CO and EG testing procedures will be

$$e_t = [\alpha(L) - \gamma(L)\lambda]' u_t + \epsilon_t$$

which obviously need not be white noise. Indeed, in general it will follow a moving average (MA) process, whose serial correlation could be accounted for by means of the semi-parametric corrections proposed by Phillips and Ouliaris (1990). It is known, however, that when the root of such MA processes are close to being the unit circle, these tests may suffer from severe size distortions; cf. Schwert (1989). This problem does not arise when using the ECM statistics.

Finally, the possibility of cointegrated regressors may arise as a practical matter (see Granger and Lee, 1990). If x_t is correcting the errors of cointegrating relationships involving only x_{t-1} , then weak exogeneity still holds (see Hunter, 1990). In that case, given that the cointegration vector does not include y_t , the proposed test is still applicable, except that the dimension of B_u in the ECM tests will be smaller than in the unrestricted version of the tests. Thus, using the critical values for the latter type of tests will lead to a conservative test. If, on the other hand, the cointegrating vector linking the x_t 's is known, then to achieve similarity future values of the $I(0)$ cointegrating error may have to be added to the regression model in (11), choosing a value of k corresponding to the number of non-cointegrated regressors in Table 1.

6. FINITE SAMPLE EVIDENCE

To provide finite sample evidence on the advantages of the ECM test-statistics in comparison with the CO and EG tests, a small set of Monte-Carlo experiments were conducted with (1) and (2) as the DGP, using 25,000 replications generated in GAUSS486. A single exogenous regressor, i.e., $k=1$, is used for illustrative purposes. Data were generated with the normalisation $\sigma_\epsilon=1$, without loss of generality, with three parameters (s , α , β) and the sample size T as experimental design variables. In this study we choose

$$\begin{aligned} s &= (0.05, 1, 5, 20) \\ \alpha &= (0.1, 0.9) \\ \beta &= (-0.05, -0.10 \text{ [cointegration in both cases]}) \\ T &= 100 \end{aligned}$$

The implied range of the *signal-to-noise* ratio is broad, including values potentially favourable and unfavourable for the relative power comparisons among the different tests. In order to simplify the analysis, the value of the cointegrating slope λ was fixed equal to 1 under the alternative hypothesis of cointegration. The choices of the short-run coefficient (α) attempt to capture a smaller ($\alpha=0.1$) and a similar value ($\alpha=0.9$) to the one chosen for λ ; the closer α and λ are, the closer the common factor restriction to be verified will be. Combining the values of α and λ with those for s we obtain a wide range of values q , ranging from 0.005 to 18.

Table 2 reports the power of the three tests for the selected range of values for α and s , when $\beta=-0.05$ and $\beta=-0.10$. To control for finite sample biases, critical values were simulated under the null hypothesis H_0 : $\beta=0$, and the reported powers are size-

adjusted. The results seem to be consistent with the discussion in section 3. When q is low and c is small relative to T , $c=-5$ ($\beta = -0.05$), the ECM test, both in its normalised bias t-ratio versions, seems to be slightly less powerful than the CO test, reflecting the problem of *dimensionality* stressed by Hansen (1990). However, as q increases, either because α becomes different from λ or because s rises, the ECM test becomes the most powerful. Furthermore, in agreement with the degeneration of the limit distributions of the coefficient version of the tests, their absolute power decreases as q increases. This is clearly not the case when we examine the t-ratio version of the tests, where the ECM test shifts its asymptotic distribution to the left so as to achieve maximum power. For example, an extreme case is when $c = -5$ ($\beta = -0.05$), $\alpha = 0.1$ and $s = 20$, where the t-ratio version of the ECM test rejects 100% of the time, while the CO test almost does not reject at all. As regards the EG test, the results indicate that its power also decreases as q increases, though at a lower rate than the power of the CO test. In agreement with the results in Banerjee *et al.* (1986), it turns out to have lower power than the ECM test, even when q is small, since in contrast to the CO test, the EG test is not dimension-invariant.

Finally, although our testing procedure is designed in a single equation framework, a comparison with Johansen's (1991) procedure would be helpful. Indeed, the ECM procedure is a special case of Johansen's for a system in which the cointegrating vectors appear only in the equation of interest. Although an extensive study on the performance of Johansen's test is beyond the scope of this paper, we have carried out a small Monte-Carlo study for the case $\beta = -0.10$ in the bivariate system consisting of equations (8) and (2). The error variances have been normalised to unity, yielding a covariance $\rho = (1+q^2)^{-1/2}q$. The trace statistic $LR(0)$ is asymptotically equal to $T\hat{\lambda}_1 + T\hat{\lambda}_2$ where $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are eigenvalues computed from some characteristic equation such as (2.11) in Johansen (1991). Suppose $\hat{\lambda}_1 > \hat{\lambda}_2$. Johansen (1991) shows that if there is a unique cointegrating vector, $T\hat{\lambda}_2$ has a asymptotic distribution while $\hat{\lambda}_1$ converges to a positive constant. It is easy to see than in our model

$$\hat{\lambda}_1 \rightarrow \lambda_1 = [1 - (1-\rho^2) (2+\beta)/\beta]^{-1} \quad (12)$$

This in turn implies for a fixed T , $LR(0)$ would (correctly) reject the null hypothesis on non-cointegration more easily if $|\rho|$ is larger and that the smaller is β rejection would be harder. Using the 5% critical value in Osterwald-Lenum (1992) with $T = 100$, we find that when $s = 1$, the rejection rate of $LR(0)$ is 46% (16%) when $\alpha = 0.1$ (0.9). When $s = 5$, they turn out to be 91% (18%). Further experiments, available upon request, show that

for high values of S the $LR(0)$ test performs slightly worse than the ECM t-test and that the behaviour is much worse for low values of S .

7. CONCLUSIONS

Testing for cointegration has become an important facet of empirical analysis of economic time series in recent years and various tests are being used. In this paper we propose a new test, denoted as ECM test, in a single equation framework. The limit distribution of this test, both in its "normalised version" and t-ratio versions, does not depend upon nuisance parameters but it does depend on the dimension of the system. Critical values are therefore provided. Its power properties are compared to those of other popular tests of cointegration. Specifically, we concentrate on the CO and EG testing procedures. The CO test is dimension-invariant whereas the EG test is not. However, both tests impose possibly invalid common factor restrictions in the estimation underlying the tests. We show that when those restrictions are invalid, the power properties of the CO and EG may be very poor in comparison to the ECM test, which does not impose those restrictions.

Moreover, as a by-product of the analysis, we show that the t-ratio form of the ECM test may be preferable to the normalised bias form, under the alternative hypothesis of cointegration, when the common factor restrictions do not hold. The results are obtained for a simple DGP and then shown to extend to more general cases.

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APPENDIX

The analysis contained in this appendix draws on a number of well known results in Phillips (1987, 1988) and Phillips and Ouliaris (1990). Under the null hypothesis of non-cointegration the DGP(H_0) is given by

$$\begin{aligned} \Delta y_t &= \alpha \Delta x_t + \epsilon_t \\ \Delta x_t &= u_t \end{aligned} \quad \text{with} \quad \begin{pmatrix} \epsilon_t \\ u_t \end{pmatrix} \sim \text{iid} \quad \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \Sigma_u \end{pmatrix} \right]$$

The results do not depend on the initialisation, so let us define $S_{\epsilon t} = \sum_{i=1}^t \epsilon_i$ and $S_{ut} = \sum_{i=1}^t u_i$, where the sums run from 1 to t , and S_ϵ and S_u are $(T \times 1)$ and $(T \times k)$ matrices of observations on $S_{\epsilon t}$ and S_{ut} , respectively. Note that $x = S_u$ and $y = S_u \alpha + S_\epsilon$. Let M_t be the projection matrix $M_t = I - x_{-1}(x_{-1}'x_{-1})^{-1}x_{-1}'$.

Then, the following set of asymptotic results (R1) will be used in the proofs:

- (a) $T^{-2} S_{\epsilon-1}' M_t S_{\epsilon-1} \Rightarrow \sigma_\epsilon^2 \int \bar{B}_\epsilon^2$
- (b) $T^{-1} S_{\epsilon-1}' M_t \epsilon \Rightarrow \sigma_\epsilon^2 \int \bar{B}_\epsilon dB_\epsilon$
- (c) $T_{-1} S_{\epsilon-1}' M_t \epsilon \rightarrow 0$
- (d) $T^{-1} \sum_1^T \epsilon_t u_t \rightarrow 0$

where $\bar{B}_\epsilon = B_\epsilon - (\int B_\epsilon B_\epsilon') \cdot (\int B_\epsilon B_\epsilon')^{-1} B_\epsilon$

Under the local alternative hypothesis of near non-cointegration the DGP (H_{1a}) is given by

$$\Delta z_t = \beta z_{t-1} + \epsilon_t$$

$$\Delta x_t = u_t$$

with $\beta = -T^{-1}c$; $z_t = y_t - \lambda'x_t$, $e_t = (\alpha - \lambda)'u_t + \epsilon_t$ and z and e are $(T \times 1)$ vectors of observations on z_t and e_t .

In this case, the following additional asymptotic results (R2) are used

- (a) $T^{-2} z'z \Rightarrow \sigma_e^2 \int K_e^2$
- (b) $T^{-2} z' M_t z \Rightarrow \sigma_e^2 \int \bar{K}_e^2$
- (c) $T^{-2} z_{-1}' M_t \epsilon \Rightarrow \sigma_e \sigma_\epsilon \int \bar{K}_e dB_\epsilon$

where $\bar{K}_e = K_e - (\int B_\epsilon K_e') \cdot (\int B_\epsilon B_\epsilon')^{-1} B_\epsilon$

PROOF OF PROPOSITION 1. Let V be a $(T \times 2k)$ matrix of observations on $v_t = (\Delta x_t', x_{t-1}')'$ and x_{t-1} and Δx be $(T \times k)$ matrices of observations on x_t and x_{t-1} , respectively. Define the projection matrix $M = I - V(V'V)^{-1}V'$ such that, by partitioned inverses, $M = M_1 - M_1 \Delta x (\Delta x' M_1 \Delta x)^{-1} \Delta x' M_1$.

Then, $\hat{\beta}_F$ is computed such that

$$\begin{aligned} T \hat{\beta}_F &= [T^{-2} y_{t-1}' M y_{t-1}]^{-1} [T^{-1} y_{t-1}' M \Delta y] \\ &= [T^{-2} S_{\epsilon-1}' M S_{\epsilon-1}]^{-1} [T^{-1} S_{\epsilon-1}' M \epsilon] \end{aligned}$$

since $y = S_u \alpha + S_\epsilon$, $\Delta y = u \alpha + \epsilon$ and M is orthogonal to x_{t-1} and Δx . Using parts (a) to (d) of (R1) and the relationship between M and M_1 , we have

$$\begin{aligned} T^{-2} S_{\epsilon-1}' M S_{\epsilon-1} &= [T^{-2} S_{\epsilon-1}' M_1 S_{\epsilon-1}] - T^{-1} [T^{-1} S_{\epsilon-1}' M_1 u] [T^{-1} u' M_1 u]^{-1} [T^{-1} u' M_1 S_{\epsilon-1}] \\ &= T^{-2} S_{\epsilon-1}' M_1 S_{\epsilon-1} + o_p(1) \end{aligned}$$

and

$$\begin{aligned} T^{-1} S_{\epsilon-1}' M \epsilon &= [T^{-1} S_{\epsilon-1}' M_1 \epsilon] - [T^{-1} S_{\epsilon-1}' M_1 u] [T^{-1} u' M_1 u]^{-1} [T^{-1} u' M_1 \epsilon] \\ &= T^{-1} S_{\epsilon-1}' M_1 \epsilon + o_p(1) \end{aligned}$$

given (e) in (R1)

Next, using the limit distributions in (a) and (b) in (R1) yields the required result

$$T \hat{\beta}_F = [S_{\epsilon-1}' M_1 S_{\epsilon-1}]^{-1} S_{\epsilon-1}' M_1 \epsilon + o_p(1) \Rightarrow (\int \bar{B}_\epsilon^2)^{-1} \int \bar{B}_\epsilon dB_\epsilon \quad (\text{A.1})$$

To prove that $\hat{\sigma}_\epsilon^2 \rightarrow \sigma_\epsilon^2$, define P as the $(T \times (2k+1))$ matrix of observations on $(\Delta x_t', x_{t-1}', y_{t-1})'$ and the projection matrix $M_p = I - P(P'P)^{-1}P'$. Then

$$\begin{aligned} \hat{\sigma}_\epsilon^2 &= T^{-1} \epsilon' M_p \epsilon = T^{-1} \epsilon' \epsilon - T^{-1} [T^{-1} \epsilon' P] [T^{-2} P' P]^{-1} [T^{-1} P' \epsilon] \\ &= T^{-1} \epsilon' \epsilon + o_p(1) \rightarrow \sigma_\epsilon^2 \end{aligned} \quad (\text{A.2})$$

From (A.1) and (A.2), the distribution of the t-ratio follows along the same lines, leading to the required results.

PROOF OF PROPOSITION 2

Let $z_t = y_t - \lambda' x_t$ and $\hat{z}_t = y_t - \hat{\alpha}' x_t$ where $\hat{\alpha}$ is the least-squares estimator of α in (1'). Then

$$z_t = \hat{z}_t + (\hat{\alpha} - \lambda)' x_t = \hat{z}_t + (\alpha - \lambda)' x_t + o_p(1)$$

since $\hat{\alpha} \rightarrow \alpha$ at rate $T^{-1/2}$

Then

$$T\hat{\beta}_E = [T^{-2}Y_{-1}'MY_{-1}]^{-1} [T^{-1}Y_{-1}'M\Delta y] = [T^{-2}Z_{-1}'MZ_{-1}]^{-1} [T^{-1}Z_{-1}'M\Delta y] \quad (\text{A.3})$$

$$= T\beta + [T^{-2}Z_{-1}'M_1Z_{-1}]^{-1} T^{-1}Z_{-1}'M_1\epsilon + o_p(1)$$

since M is orthogonal to x_{-1} and Δx , and the limit distribution of $(T^{-2}Z_{-1}'MZ_{-1})$ is equal to the limit distribution of $[T^{-2}Z_{-1}'M_1Z_{-1}]$, following the same arguments as in the proof for Proposition 2.

Finally, using $T\beta = -c$ and substituting results (a) to (c) of (R2) into (A.3) yields the required results. Since $\hat{\sigma}_\epsilon \rightarrow \sigma_\epsilon$ and $\hat{\sigma}_e \rightarrow \sigma_e$, the proof for the limit distribution of the t-ratio follows along similar lines, leading to the required results.

PROOF OF PROPOSITION 3

For $k = 1$, from the limit distributions in Proposition 2, we have

$$T\hat{\beta}_E \Rightarrow -c + (\sigma_\epsilon/\sigma_e) [\int \bar{K}_e^2]^{-1} \int \bar{K}_e dB_\epsilon$$

Since $\sigma_\epsilon/\sigma_e = (1+q^2)^{-1/2}$, as $q \uparrow \infty$, we have that $(\sigma_\epsilon/\sigma_e) \uparrow 0$ and

$$T\hat{\beta}_E \Rightarrow -c + o_p(q^{-1})$$

Furthermore, since $\hat{\sigma}_\epsilon \rightarrow \sigma_\epsilon$ and $\hat{\sigma}_e \rightarrow \sigma_e$, the proof for the limit distribution of the t-ratio proceeds along similar lines.

Table 1

Critical values of the (t-ratio) ECM Test
 Different number of regressors

		Size			
T		0.01	0.05	0.10	0.25
A. (with constant)					
(k=1)	25	-4.12	-3.35	-2.95	-2.36
	50	-3.94	-3.28	-2.93	-2.38
	100	-3.92	-3.27	-2.94	-2.40
	500	-3.82	-3.23	-2.90	-2.40
	∞	-3.78	-3.19	-2.89	-2.41
(k=2)	25	-4.53	-3.64	-3.24	-2.60
	50	-4.29	-3.57	-3.20	-2.63
	100	-4.22	-3.56	-3.22	-2.67
	500	-4.11	-3.50	-3.10	-2.66
	∞	-4.06	-3.48	-3.19	-2.65
(k=3)	25	-4.92	-3.91	-3.46	-2.76
	50	-4.59	-3.82	-3.45	-2.84
	100	-4.49	-3.82	-3.47	-2.90
	500	-4.47	-3.77	-3.45	-2.90
	∞	-4.46	-3.74	-3.42	-2.89
(k=4)	25	-5.27	-4.18	-3.68	-2.90
	50	-4.85	-4.05	-3.64	-3.03
	100	-4.71	-4.03	-3.67	-3.10
	500	-4.62	-3.99	-3.67	-3.11
	∞	-4.57	-3.97	-3.66	-3.10
(k=5)	25	-5.53	-4.46	-3.82	-2.99
	50	-5.04	-4.43	-3.82	-3.18
	100	-4.92	-4.30	-3.85	-3.28
	500	-4.81	-4.39	-3.86	-3.32
	∞	-4.70	-4.27	-3.82	-3.29

B. (with constant and trend)

(k=1)	25	-4.77	-3.89	-3.48	-2.88
	50	-4.48	-3.78	-3.44	-2.92
	100	-4.35	-3.75	-3.43	-2.91
	500	-4.30	-3.71	-3.41	-2.91
	∞	-4.27	-3.69	-3.39	-2.89
(k=2)	25	-5.12	-4.18	-3.72	-3.04
	50	-4.76	-4.04	-3.66	-3.09
	100	-4.60	-3.98	-3.66	-3.11
	500	-4.54	-3.94	-3.64	-3.11
	∞	-4.51	-3.91	-3.62	-3.10
(k=3)	25	-5.42	-4.39	-3.89	-3.16
	50	-5.04	-4.25	-3.86	-3.25
	100	-4.86	-4.19	-3.86	-3.30
	500	-4.76	-4.15	-3.84	-3.31
	∞	-4.72	-4.12	-3.82	-3.29
(k=4)	25	-5.79	-4.56	-4.04	-3.26
	50	-5.21	-4.43	-4.03	-3.39
	100	-5.07	-4.38	-4.02	-3.46
	500	-4.93	-4.34	-4.02	-3.47
	∞	-4.89	-4.30	-4.00	-3.45
(k=5)	25	-6.18	-4.76	-4.16	-3.31
	50	-5.37	-4.60	-4.19	-3.53
	100	-5.24	-4.55	-4.19	-3.66
	500	-5.15	-4.54	-4.20	-3.69
	∞	-5.11	-4.52	-4.18	-3.67

Table 2
Size Adjusted Powers of 5% Tests
(percentages)

$\beta = -0.05$					
Test		s=0.05	s=1.00	s=5.00	s=20.00
$\alpha=0.1$	C-O	30 (30)	8 (7)	0 (0)	0 (0)
	ECM	22 (18)	14 (23)	0 (88)	0 (100)
	EG	14 (15)	11 (11)	5 (5)	4 (4)
$\alpha=0.9$	C-O	30 (30)	28 (28)	16 (16)	1 (1)
	ECM	21 (17)	21 (17)	18 (19)	5 (48)
	EG	14 (14)	13 (14)	12 (12)	7 (7)
$\beta = -0.10$					
$\alpha=0.1$	C-O	69 (68)	8 (8)	0 (0)	0 (0)
	ECM	53 (54)	44 (67)	8 (100)	0 (100)
	EG	36 (36)	30 (30)	18 (18)	17 (17)
$\alpha=0.9$	C-O	70 (70)	67 (67)	27 (26)	1 (1)
	ECM	53 (54)	53 (55)	51 (53)	30 (94)
	EG	37 (37)	37 (38)	34 (35)	22 (23)

Note: Rejection rates for the t-ratio version of the tests are given in parenthesis.